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Minimize System Cost by Choosing Optimal Subsystem Reliability and Redundancy

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SUMMARY & CONCLUSIONS

The basic question which we address in this paper is how to choose among competing subsystems. This paper utilizes both reliabilities and costs to find the subsystems with the lowest overall expected cost. The paper begins by reviewing some of the concepts of expected value. We then address the problem of choosing among several competing subsystems. These concepts are then applied to k-out-of-n: G subsystems. We illustrate the use of the authors' basic program in viewing a range of possible solutions for several different examples. We then discuss the implications of various solutions in these examples.

1. *INTRODUCTION*

How does a design engineer or manager choose between a power subsystem with .990 reliability and a more costly subsystem with .995 reliability? When is the increased cost of a more reliable subsystem justified?

High reliability is not necessarily an end in itself. High reliability may be desirable in order to reduce the statistically expected cost due to a subsystem failure. However, this may not be the wisest use of funds since the expected cost due to subsystem failure is not the only cost involved. To answer this question the engineer needs to consider not only the cost of the subsystem but also to assess the costs that would occur if the subsystem fails. These costs are weighted by the probability of their occurrence to yield the expected cost. We therefore minimize the total of the <u>two costs</u>, i.e., the total of the cost of the subsystem plus the expected cost due to subsystem failure.

Since this problem involves probabilistic decision making, we'll first review some aspects of probability and expected value. We'll then apply these procedures to a simple situation of choosing between two or more competing subsystems and show how to choose the best subsystem. These principles will then be applied to choosing from among various k-out-of-n:G subsystems. The authors have written a basic program (CARRAC) which enables the engineer to explore and graph various options. We'll illustrate the use of this program with several different cost models.

<u>Notation</u>

- n number of modules in the subsystem
- k minimum number of good modules for the subsystem to be good
- r reliability of the whole system for other than failure of the subsystem
- c_1 loss due to failure of the subsystem
- c_3 cost of a one module subsystem capable of full output
- c_4 cost of a module in a k-out-of-n:G subsystem when k is fixed
- r_{si} reliability of subsystem i, i = 1, 2, ...
- $g(k)$ function relating cost of subsystem to the number of modules in subsystem
- p probability that a <u>module</u> is good
- q probability that a <u>module</u> fails or 1-p
- C the total of the cost of the subsystem itself plus the expected cost due to subsystem failure

2. *EXPECTED VALUE*

Since much of the paper is founded upon the idea of expected value or expected cost, we'll review some of the fundamental uses of this concept in decision-making applications. Suppose that you may choose between actions A and B. In this example, action A always results in a \$1000 return to you. Then A has a value of \$1000 and we can say that the expected value of A, E(A), is \$1000. Action B, on the other hand, results in a return to you of either \$500, outcome B_1, or \$1500, outcome B_2. This return is a random variable which depends upon circumstances beyond your control. The choices which you face are outlined in figure 1.

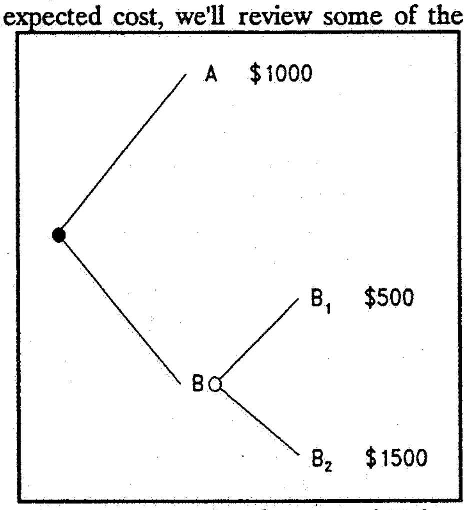

Figure 1 Example of Expected Value.

If B_1 and B_2 are equally likely, i.e., $Pr(B_1) = Pr(B_2) = .5$

(where Pr means "probability of"), then $E(B) = \$500xPr(B_1) + \$1500xPr(B_2) = \$500(.5) + \$1500(.5) = \$1000$. If you use expected value as your criterion, then you would be indifferent as to choice A or B, since both have an expected value of \$1000. Also note that, although B has an expected value of \$1000, you never receive \$1000. Half of the time you receive \$500 and half of the time you receive \$1500.

Now suppose that the probabilities of B_1 and B_2 are .4 and .6, respectively. Then $E(B) = \$500(.4) + \$1500(.6) = \$1100$. If you use expected value as your criterion, you would choose B over A, since it has the higher expected value. In unusual circumstances, such as the need to repay \$1000, you might choose A over B, even though A has the lower expected value. For these types of circumstances, we say that the certain return of \$1000 has a higher expected utility to you than the expected utility associated with an expected value of \$1100, where the return can be either \$500 or \$1500. For unusual circumstances, the procedures outlined in this paper can be applied using expected utility rather than expected value. For a more detailed discussion of utility, see [1].

Suppose instead that action A results in a cost of \$1000 while action B results in a cost of either \$500 or \$1500. We could, in a manner similar to that above, analyze actions A and B in terms of their expected costs. Our objective would be to minimize expected cost. Throughout the remainder of this paper we will use expected value or expected cost as our criterion.

3. *BALANCING TWO COSTS*

We will be using expected value as our criterion, namely the expected cost due to subsystem failure, shown as E{cost due to subsystem failure}. As with all expected values, this number depends upon both the dollar cost and the probability of its occurrence. Let c_1 be the dollar cost due to failure of the subsystem, including all costs incurred by subsystem failure (but not the cost of the subsystem itself). This number could be the entire cost of the main system (even greater in some circumstances) if failure of the subsystem resulted in complete failure of the main system. In other instances c_1 could be less than the cost of the subsystem, e.g., cost of the subsystem resulted in only a partial failure of the main system.

Now the expected cost due to subsystem failure is c_1 times the probability that this cost will be experienced. If the main system fails (for other than failure of the subsystem) then there is no cost due to subsystem failure. For example, if we're considering a power subsystem in a rocket, the rocket may explode on the launch pad due to a fuel problem. Even if the power subsystem failed in flight, we would not experience this failure. Let r be the reliability of the main system (for other than failure of the subsystem) and let r_s be the reliability of the subsystem. Then E{cost due to subsystem failure} = c_1Pr{subsystem failure | mainsystem good} Pr{main system good} = $c_1(1-r_s)r = rc_1(1-r_s)$.

We can minimize this expected cost by building a subsystem with an extremely low probability of failure, i.e., a subsystem with extremely high reliability. In this situation it is not clear that we should build the most reliable subsystem possible since this will minimize only the expected cost due to subsystem failure but does not consider the cost of the building the subsystem itself. To make this decision, we should not consider the two costs separately. <u>We therefore minimize the total of the two costs, i.e., the total of the cost of the subsystem plus the expected cost due to subsystem failure</u>. The total quantity to be minimized is given by

C ≡ cost of the subsystem + E{cost due to subsystem failure}

= cost of the subsystem + $rc_1(1-r_s)$.

<u>In minimizing C, we see that we are balancing the cost of the subsystem itself and the expected cost due to subsystem failure</u>.

4. *SELECTING THE BETTER SUBSYSTEM*

As an example, suppose that we have two possible subsystems under consideration. Subsystem 1, which costs \$200,000, has a .97 reliability. Subsystem 2, with a cost of \$100,000, has a .94 reliability. Without further information and analysis, there is no clear "best" subsystem, and the choice is often based upon the amount budgeted for the subsystem.

For further analysis, let's assume that the two subsystems under consideration will be part of a main system which has a reliability (exclusive of the subsystem under consideration) of $r = .96$. We'll further assume that failure of the subsystem will result in a cost of $c_1 = \$10,000,000$. Let us first look at the E{cost due to subsystem failure} for each of the two subsystems. For subsystem 1,

E{cost due to subsystem failure} = rc_1Pr{subsystem failure} = $rc_1(1-r_{s1})$ = .96x\$10,000,000x.03 = \$288,000.

For subsystem 2,

E{cost due to subsystem failure} = $rc_1(1-r_{s2})$ = .96x\$10,000,000x.06 = \$576,000.

Since subsystem 2 is less reliable than subsystem 1 it has a higher expected cost. However, since 2 is also less expensive, we need to compare the overall expected cost, C, for 1 and for 2. For subsystem 1,

C_{s1} = \$200,000 + \$288,000 = \$488,000.

For subsystem 2,

C_{s2} = \$100,000 + \$576,000 = \$676,000.

Since $C_{s1} < C_{s2}$, we select subsystem 1 over subsystem 2.

5. *K-OUT-OF-N:G SUBSYSTEMS*

We'll now direct our attention to a specific type of subsystem, called a k-out-of-n:G subsystem. Such a subsystem has n modules, of which k are required to be good for the subsystem to be good. As an example consider the situation where the engineer has a certain power requirement. He may meet this requirement by having one large power module, two smaller modules, etc. The number of modules required is called k. For example, the engineer

may decide that k = 4. Then each module is 1/4 of the full required power. Therefore, the subsystem must have 4 or more modules for the full required power. The number of modules used in the subsystem is called n. For example, an n = 6 and k = 4 subsystem would have 6 modules each of 1/4 power and thus would have the output capability of 1.5 times the required power. The engineer chooses n and k. Selection of the different values of n and k results in different subsystems, each with different costs and reliabilities. Since each n and k yields different subsystems with different costs, we can choose the subsystem (the n and k) which will minimize cost C.

Assumptions for k-out-of-n:G subsystems

1. The probability of failure of any module in the system is not affected by the failure of any other module, i.e, the modules are s-independent.
2. There is a k-out-of-n:G subsystem where each of the modules has the same probability of success.
3. Failure of the subsystem results in a loss of c_1; c_1 includes all losses incurred due to subsystem failure but not the cost of the subsystem itself.

5.1 *MODEL 1*

For model 1 we assume that k is fixed and that each module costs c_4. Now E{cost due to subsystem failure} = rc_1Pr{subsystem failure} = rc_1 Pr{X<k} = rc_1 binf(k-1; p,n). Since C = cost of subsystem + E{cost due to subsystem failure}, then C = nc_4 + rc_1 binf(k-1;p,n).

The authors have written a Basic program (CARRAC) to find the n which minimizes C. Additionally CARRAC will graph C as a function of either p or c_1 and graphs the best subsystems (i.e. the ones with the lowest C's) over ranges of p or c_1. This allows you to not only select the best subsystem for a particular value of p or c_1 but also to view what happens to C for nearby values of p or c_1.

As an example, consider the situation when k = 1, that is only one module is required to be good for the subsystem to be good. The reliability of this single module is estimated to be .95 (p = .95). Let the reliability of the system for other than failure of the subsystem be .9 (r = .9). The cost of one module is 1 (c_4 = 1) million dollars. The cost due to failure of this subsystem is 10 (c_1 = 10) million dollars.

Figure 2 shows a plot of C for .79 < p < .99 and n's of 1 through 4. When the reliability of a single module is p = .95, then the n = 1 subsystem has the lowest value of C. Therefore the best subsystem is the one with no spares. We see from figure 2 that the n = 1 subsystem has the lowest value of C for any p > .87. If p < .87, then n = 2 (one spare) has the lowest value of C.

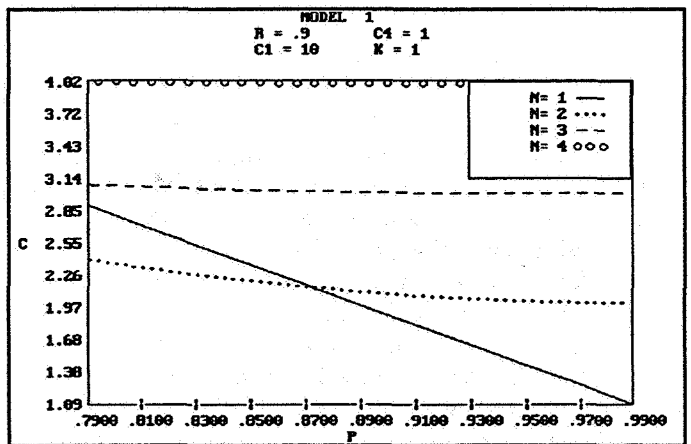

Figure 2 {Cost of Subsystem plus Expected Cost Due to Subsystem Failure} vs Reliability of a Single Subsystem Module.

Now suppose instead that c_1 (cost due to failure of the subsystem) is 50. Figure 3 shows the plot of C for c_1 = 50. We first note that if p = .95, then the n = 2 subsystem (one spare) is the best. Comparing figures 2 and 3 (at p = .95) we see that the larger value of c_1 (in figure 3) requires a larger value of n. In general, if the cost of subsystem failure increases, then more redundancy is required. If .83 < p < .98, figure 3 shows that the n = 2 subsystem is best. If p < .83 then still more redundancy (n=3) is required. If p > .98, then no redundancy (n=1) is required.

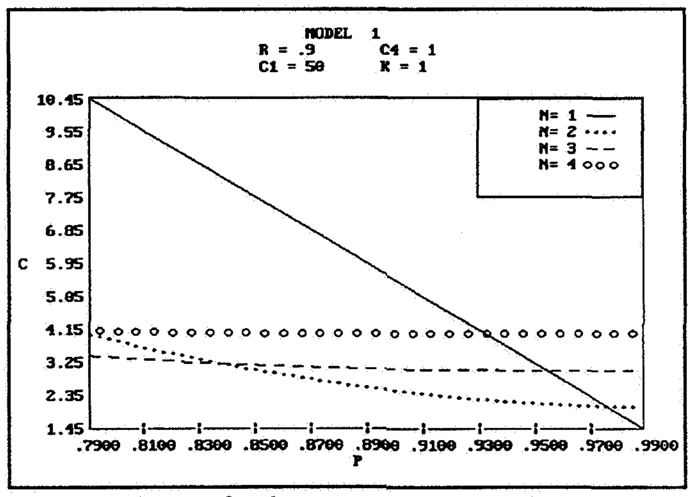

Figure 3 {Cost of Subsystem plus Expected Cost Due to Subsystem Failure} vs Reliability of a Single Subsystem Module.

5.2 *MODEL 2*

Assumptions

Same as model 1 except:

1. We are free to choose k in our subsystem.
2. The cost of a k-module subsystem is c_3 g(k).
3. Each module in the subsystem costs c_3g(k)/k. Since there are n modules in the subsystem, the cost of the subsystem = nc_3g(k)/k. Therefore C

$= \text{cost of subsystem} + E\{\text{loss due to subsystem failure}\} = nc_3g(k)/k + rc_1 \, \text{binf}(k\text{-}1; p,n)$.

We note that g(k) usually increases in k, since it is generally more expensive to have subsystems consisting of k smaller elements than to have a subsystem consisting of a single large module. As an example of model 2, suppose we are building a space electrical power subsystem. The cost due to subsystem failure, c_1, is 240. Let the reliability of the system for other than failure of the subsystem be .9 (r = .9). Suppose that the cost of building a single module capable of full power is 1 ($c_3 = 1$). A rough rule of thumb says that the cost of smaller modules for a space electrical power subsystem is proportional to the electrical power raised to the .7, i.e., $g(k) = k(1/k)^{.7}$. Therefore, a subsystem consisting of a single module capable of full power would cost $c_3g(1) = c_3 1(1/1)^{.7} = 1.0c_3$, a subsystem consisting of 2 modules, each of 1/2 power, would cost $c_3g(2) = c_3 2(1/2)^{.7} = 1.23c_3$ to build, etc. An n = 3 and k = 2

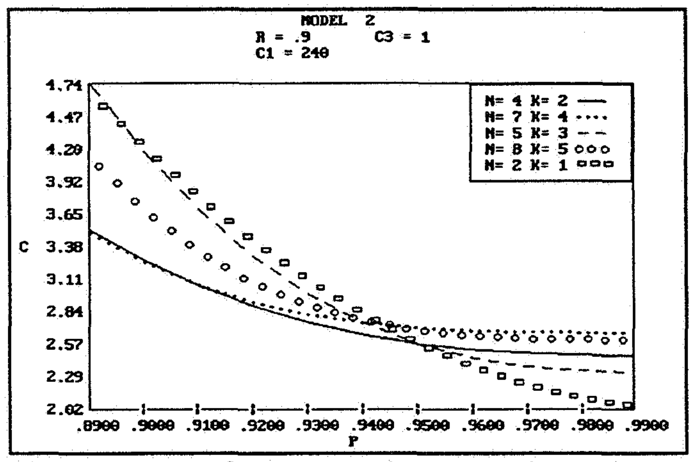

Figure 4 {Cost of Subsystem plus Expected Cost Due to Subsystem Failure} vs Reliability of a Single Subsystem Module.

subsystem, i.e., one having 3 modules each of 1/2 power, would cost $nc_3 \, g(k)/k = 3\text{x}1.23c_3/2 = 1.85c_3$ to build. An estimate of p, the reliability of an individual module, is .96. If we are unsure of this estimate, we can use CARRAC to view (figure 4) the best subsystems over p ranging from .89 to .99. From figure 4, at p = .96, the n = 2, k = 1 subsystem is best (lowest value of C). If p < .95, the n = 4, k = 2 subsystem is best. Note this is a flatter curve over the range of p, indicating a low value for C over a wide range of p.

For the example, suppose we wish to view what happens to C as c_1 varies. Possibly we are fairly confident about our estimate of p = .96 but unsure about our estimate of c_1. Figure 5 shows, if c_1 is below 310, that the n = 2, k = 1 subsystem is best. However, for $310 < c_1 < 400$, the n = 5, k = 3 subsystem is the best. For $c_1 > 400$ the n = 4, k = 2 subsystem is the best. This type of analysis can be used whenever you are unsure of c_1 and wish to consider a wider range of values.

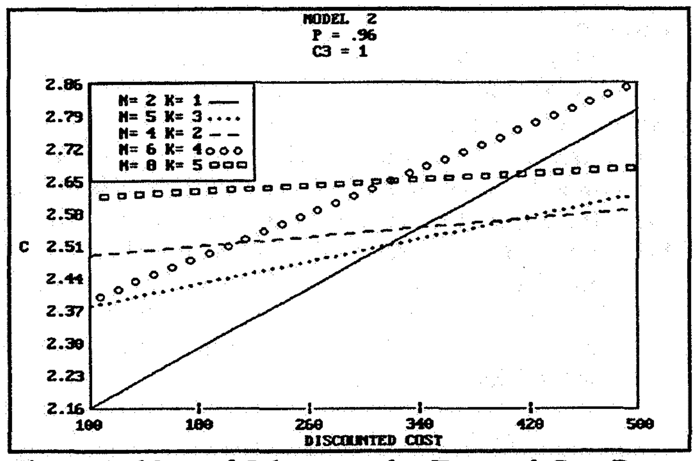

Figure 5 {Cost of Subsystem plus Expected Cost Due to Subsystem Failure} vs (Discounted) Cost of Subsystem Failure.

6. *OTHER MODELS AND THE BASIC PROGRAM*

CARRAC can be used to explore near optimal solutions for the three other cost models presented by Suich & Patterson [2,3]. These other models cover time dependency and situations with and without salvage value. The authors have sent copies of the CARRAC to selected organizations in the United States for initial testing. If you or your organization are interested in participating, please contact Richard Patterson. It is anticipated that CARRAC will be available in the future through NASA's Computer Software Management and Information Center (COSMIC).

7. *REFERENCES*

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8. *BIOGRAPHIES*

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13. ABSTRACT ***(Maximum 200 words)***

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